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Introduction to Electronics

An introduction to electronic components and a study of circuits containing such devices.

TECH



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Second-Order Transfer Functions

Introduce second-order filter transfer functions

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Previous Lesson

- Introduced cascaded first-order op-amp filters

Lesson Objectives

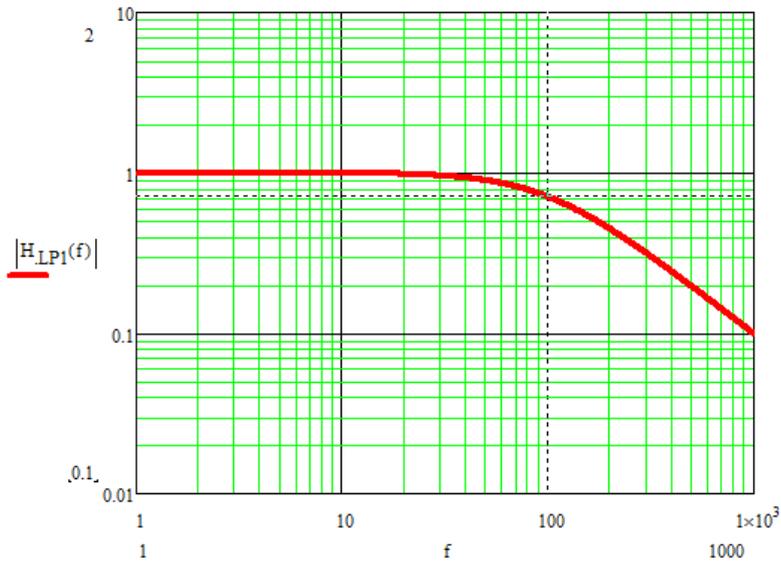
- Introduce second-order filter transfer functions
- Examine features of transfer functions

Filter Transfer Function

- Ratio of output voltage to input voltage as a function of frequency
- For any frequency, the transfer function is a complex number that indicates how the filter modifies the magnitude and phase of the input to produce the output

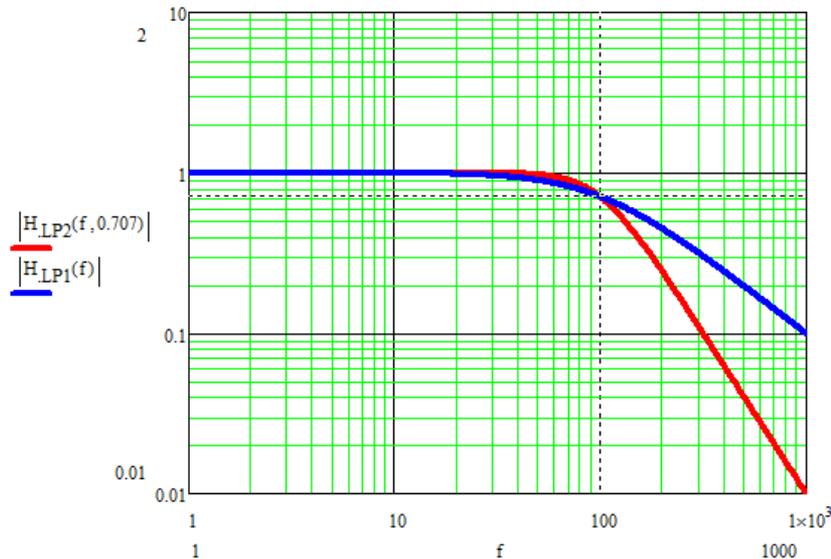
$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

First-Order Low-Pass Filter



$$H_{LP1}(f) = K \frac{1}{\frac{jf}{f_0} + 1}$$

Second-Order Low-Pass Filter

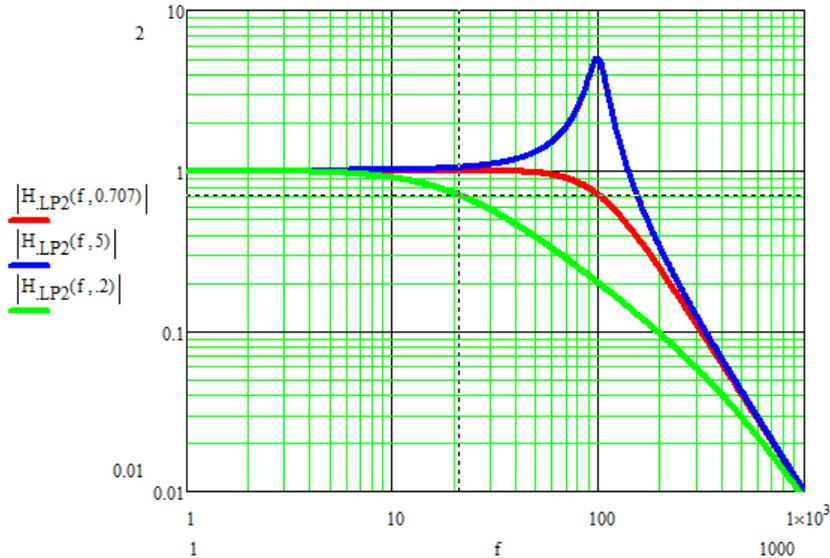


$$H_{LP1}(f) = K \frac{1}{\frac{jf}{f_0} + 1}$$

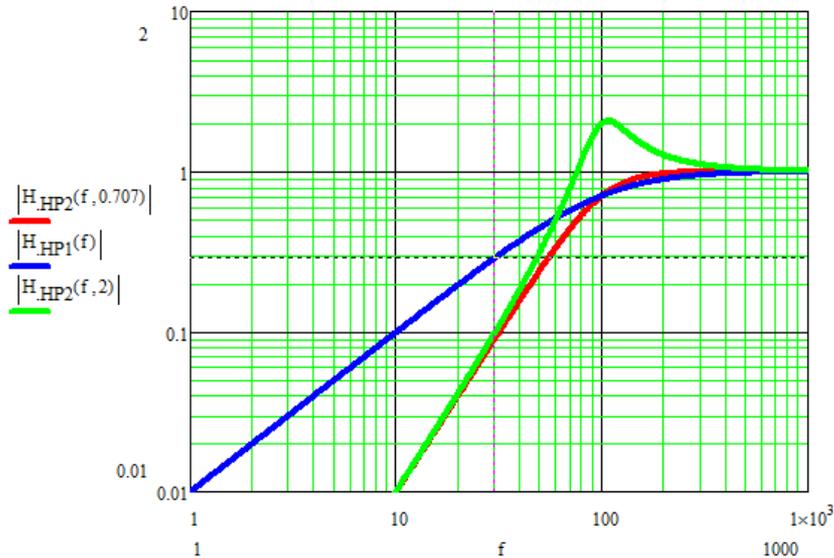
$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

Effect of Quality Factor (Q)

$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$



High-Pass Filters



$$H_{HP1}(f) = K \frac{\frac{jf}{f_0}}{\frac{jf}{f_0} + 1}$$

$$H_{LP1}(f) = K \frac{1}{\frac{jf}{f_0} + 1}$$

$$H_{HP2}(f) = K \frac{\left(\frac{jf}{f_0}\right)^2}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

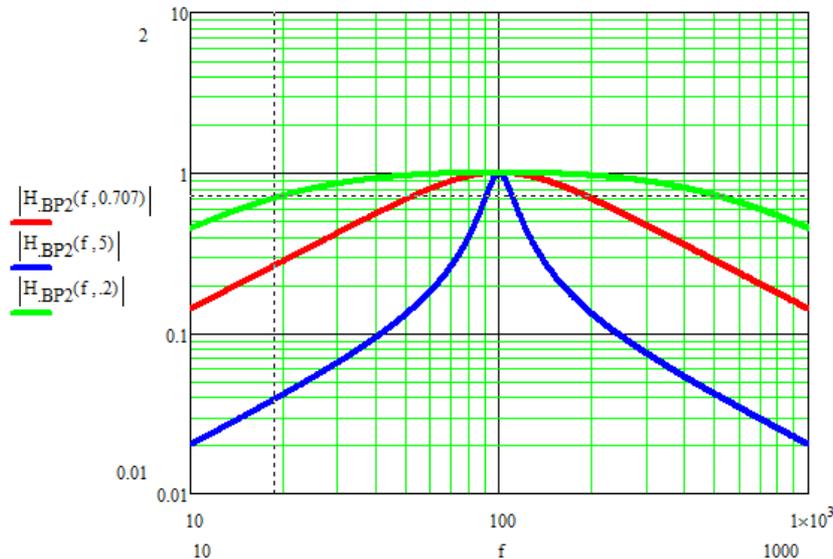
$$H_{LP2}(f) = K \frac{1}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

Band-Pass Filters

$$H_{BP2}(f) = K \frac{\frac{jf}{f_0} \frac{1}{Q}}{\left(\frac{jf}{f_0}\right)^2 + \frac{jf}{f_0} \frac{1}{Q} + 1}$$

$$Q = \frac{f_0}{BW}$$

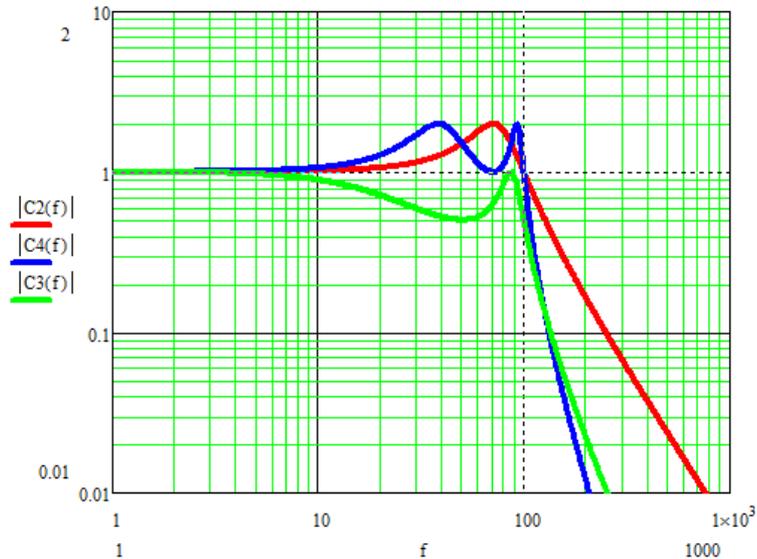
$$BW = \text{Bandwidth} = f_u - f_l$$



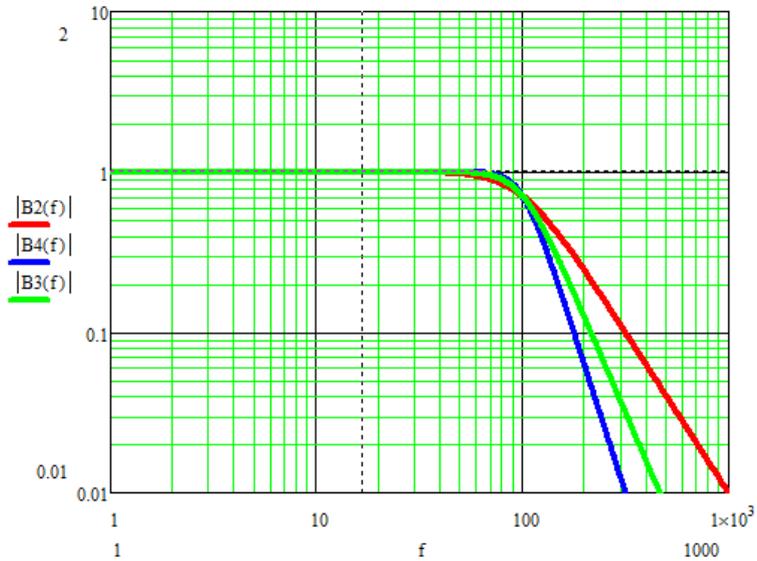
Butterworth and Chebyshev

- Types of transfer functions
- For second-order filters, the type is determined by the Q value
- $Q = 1/\sqrt{2}$ Butterworth (Maximally Flat)
- $Q > 1/\sqrt{2}$ Chebyshev

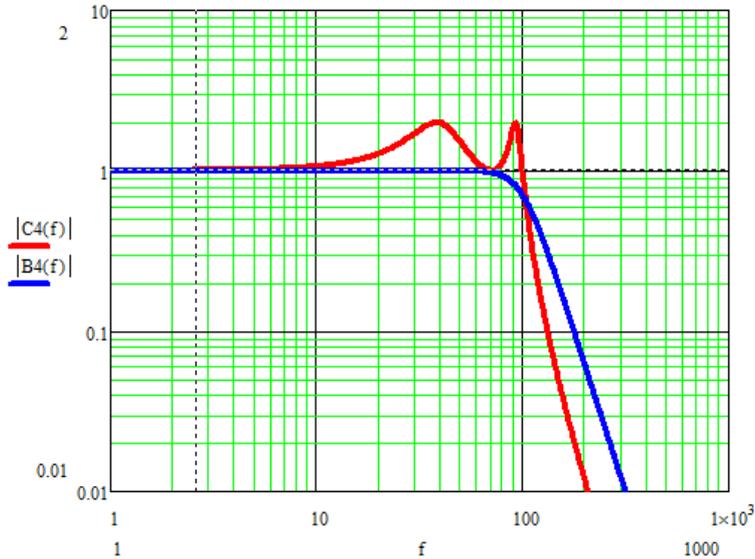
Chebyshev Filters



Butterworth Filters



Fourth-Order Butterworth vs. Chebyshev



Summary

- ⦿ Introduced second-order transfer functions
- ⦿ Examined features of transfer functions

Next Lesson

- Op-Amp Second-Order Filter Circuits